Role of non-magnetic disorder in a doped U(1) spin liquid

Ki-Seok Kim
Korea Institute of Advanced Study, Seoul 130-012, Korea
(Dated: February 2, 2008)

Recently we investigated a role of non-magnetic disorder on the stability of a $U(1)$ spin liquid ($U1SL$) [cond-mat/0407151; Phys. Rev. B (R) accepted]. In the present paper we examine an effect of the non-magnetic disorder on a doped $U1SL$. In a recent study [cond-mat/0408236] we have shown that the doped $U1SL$ shows deconfined massive spinon excitations in the superconducting phase as a result of holon condensation. We find that the massive spinon is trapped by the non-magnetic disorder. Owing to the localized spin the non-magnetic disorder acts as magnetic one.

PACS numbers: 74.20.Mn, 71.23.An, 73.43.Nq, 11.10.Kk

It is now believed that non-magnetic disorder induces a local magnetic moment of $S = \frac{1}{2}$ in underdoped cuprates[1, 2, 3, 4, 5]. The localized moment is expected to result in a Curie-Weiss behavior of a magnetic susceptibility[1, 3, 5] and increase antiferromagnetic correlations[2]. Despite of these successful descriptions there exists controversy. Sachdev et al. claimed that emergence of the local moment in a non-magnetic impurity can be understood by confinement of a spinon in the underdoped cuprates[4, 6]. But the previous studies[1, 3] are based on a deconfinement phase which is described by a slave particle mean field theory. It has been correctly pointed out that in the slave boson mean field theories strong gauge fluctuations mediating interactions between the slave particles are not appropriately treated[7, 8]. Thus it is not clear that if strong gauge fluctuations are appropriately treated, the conclusions in the previous studies[1, 3] are sustained. It is necessary to treat both the gauge fluctuations and non-magnetic impurities on equal footing.

Lastly Senthil et al. claimed that instanton excitations originating from strong gauge fluctuations can be suppressed by critical fluctuations of massless Dirac fermions in the large flavor limit[9]. As a result an effective field theory to describe a spin degree of freedom in the underdoped cuprates is obtained to be QED_3 in terms of massless Dirac fermions (spinons carrying only spin $\frac{1}{2}$) interacting via non-compact U(1) gauge fields[9, 10]. The state described by this critical field theory is usually called a U(1) spin liquid ($U1SL$).

Recently we examined a role of non-magnetic disorder on the stability of the $U1SL$ using a renormalization group calculation[11]. In other words, we treated both strong gauge fluctuations and non-magnetic impurities on equal footing. In the study we found that the $U1SL$ remains stable against weak disorder. The IR fixed point of QED_3 to govern the $U1SL$ is stable against the presence of the weak non-magnetic disorder and thus the $U1SL$ is sustained. This shows that the massless Dirac spinon is not localized by the weak disorder in the $U1SL$, which is in contrast with the results of the slave particle mean field theories[1, 3].

In the present paper we investigate an effect of the non-magnetic disorder on a hole doped $U1SL$. Not only strong gauge fluctuations and non-magnetic impurities but also hole doping is treated on equal footing. Recently we showed that hole doping to the $U1SL$ can give rise to serious change in the $U1SL$[12]. In the study we claimed that condensation of holons (representing doped holes) results in a fermion (spinon) zero mode in the presence of an instanton potential. As a result instanton excitations are suppressed and thus deconfinement of the spinons and holons are expected to occur. Further, in the superconducting phase resulting from holon condensation we found that the zero mode gives rise to a mass to the Dirac fermion[12]. As a consequence the massive spinons are deconfined in the underdoped superconducting state. We call this spin state a gapped $U1SL$. This deconfinement mechanism is totally different from that in the undoped $U1SL$. In the $U1SL$ critical fluctuations of the Dirac spinons are expected to suppress the instanton excitations and thus the massless spinons are deconfined[9, 10]. We find that the massive spinon in the gapped $U1SL$ is localized in the non-magnetic disorder in contrast with the case of the gapless $U1SL$. In the critical $U1SL$ the massless spinon remains delocalized even in the presence of the weak disorder[11] as discussed above. Owing to the localized spin the non-magnetic disorder acts as magnetic one.

First we review a one dimensional hole doped Mott insulator. Reviewing this, we can understand a role of doped holes in a Mott insulator. Further, we will see that doped holes in a two dimensional doped $U1SL$ play a similar role as the case of the one dimensional doped Mott insulator. We consider the $t-J$ Hamiltonian to describe a doped Mott insulator

$$H = -t\sum_{i=1}^{N}(c_{\sigma i}^{\dagger}c_{\sigma i+1} + h.c.) + J\sum_{i=1}^{N}\mathbf{S}_i\cdot\mathbf{S}_{i+1}. \quad (1)$$

In the absence of hole doping hopping of electrons denoted by the first term is suppressed and thus the $t-J$ Hamiltonian is reduced to the Heisenberg Hamiltonian describing a quantum antiferromagnetic spin chain. Low energy physics of the quantum spin chain can be described by a non-linear σ model with a Berry phase term[13]. Utilizing CP^1 representation, one can represent the non-linear σ model in terms of bosonic spinons

interacting via compact U(1) gauge fields in the presence of the Berry phase[13]. In the case of integer spin the Berry phase is ignorable. Strong quantum fluctuations originating from low dimensionality lead the bosonic spinons to be massive[13]. The integer spin chain becomes disordered. The massive spinons are confined via a linearly increasing gauge potential in a distance which results from Maxwell kinetic energy of the gauge field[14, 15]. As a consequence mesons (spinon-antispinon bound states, here spin excitons) are expected to appear[14, 15]. In the case of half-odd integer spin the Berry phase plays a crucial role to cause destructive interference between quantum fluctuations, thus weakening quantum fluctuations. Owing to the Berry phase contribution the half-odd integer spin chain is expected to be ordered. But low dimensionality leads the system not to be ordered but to be critical[13]. As a result the spinons are massless[14, 15]. The massless spinons are expected to be deconfined[15] because critical fluctuations of the spinons can weaken gauge fluctuations via screening.

Now we consider hole doping to the spin chain. Then hopping of doped holes is admitted. Shankar showed that the doped holes are represented by massless Dirac fermions and the fermionic holes interact with the bosonic spinons via U(1) gauge fields[16]. A low energy effective field theory is obtained to be[16]

$$S = \int d^2x \Big[\frac{1}{2g}|(\partial_\mu - ia_\mu)z_\sigma|^2 + m^2|z_\sigma|^2 + \frac{u}{2}(|z_\sigma|^2)^2 - iS\epsilon_{\mu\nu}\partial_\mu a_\nu\Big] + \int d^2x \Big[\bar{\psi}_A\gamma_\mu(\partial_\mu + ia_\mu)\psi_A + \bar{\psi}_B\gamma_\mu(\partial_\mu - ia_\mu)\psi_B\Big] \quad (2)$$

Here z_σ is a bosonic spinon (spin) and $\psi_{A(B)}$, a fermionic holon (charge) in a sublattice $A(B)$. The spinons and holons interact via the compact U(1) gauge field a_μ. g^{-1} is a stiffness parameter, m^2, a mass of the spinons, and u, a strength of self-interaction[15]. A detailed derivation is given by Ref. [16]. A key question is what an effect of the massless Dirac fermions on spinon dynamics is. The massless Dirac fermions are shown to kill the Berry phase effect[17]. In order to see this we utilize a standard bosonization method[16, 17]

$$\bar{\psi}_A\gamma_\mu\partial_\mu\psi_A = \frac{1}{2}(\partial_\mu\phi_A)^2, \;\; \bar{\psi}_B\gamma_\mu\partial_\mu\psi_B = \frac{1}{2}(\partial_\mu\phi_B)^2,$$
$$\bar{\psi}_A\gamma_\mu\psi_A = \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial_\nu\phi_A, \;\; \bar{\psi}_B\gamma_\mu\psi_B = \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial_\nu\phi_B \quad (3)$$

Here ϕ_A and ϕ_B are bosonic fields in each sublattice. Inserting these into the above action Eq. (2), we obtain

$$S = \int d^2x \Big[\frac{1}{2g}|(\partial_\mu - ia_\mu)z_\sigma|^2 + m^2|z_\sigma|^2 + \frac{u}{2}(|z_\sigma|^2)^2\Big] + \int d^2x \Big[\frac{1}{2}(\partial_\mu\phi_+)^2 + \frac{1}{2}(\partial_\mu\phi_-)^2 + i\sqrt{\frac{2}{\pi}}\phi_-\epsilon_{\mu\nu}\partial_\mu a_\nu - iS\epsilon_{\mu\nu}\partial_\mu a_\nu\Big] \quad (4)$$

with $\phi_+ = \frac{1}{\sqrt{2}}(\phi_A + \phi_B)$ and $\phi_- = \frac{1}{\sqrt{2}}(\phi_A - \phi_B)$. Shifting the ϕ_- field to $\phi_- + \sqrt{\frac{\pi}{2}}S$, we can easily see that the Berry phase term is wiped out from the action. Thus half-odd integer spin chains are not distinguishable from integer spin chains. The bosonic spinon in the doped half-odd integer spin chain is expected to be massive like that in the undoped integer spin chain. But the spinons here are not confined in contrast with the case of integer spin chains[16, 17]. Integrating over the ϕ_- field, we find that the U(1) gauge field becomes massive and thus it is ignorable in the low energy limit. As a consequence the massive spinons are deconfined. A U(1) spin liquid with massive spinon excitations emerges in a doped antiferromagnetic spin chain. If we introduce an electromagnetic field A_μ, we obtain a coupling term of $i\sqrt{\frac{2}{\pi}}\phi_+\epsilon_{\mu\nu}\partial_\mu A_\nu$. Integrating over the ϕ_+ field, we obtain a mass of the electromagnetic field. This implies superconductivity in the doped spin chain, which is consistent with the result of Shankar[16]. In summary, doped holes lead a spin degree of freedom to be a gapped $U1SL$ and a charge degree of freedom to be superconducting. It should be noted that this result is exact in the low energy limit[16]. As will be seen below, these roles of the doped holes in a one dimensional Mott insulator are very similar to that in a two dimensional one.

Now we investigate a role of non-magnetic disorder in this massive $U1SL$. An impurity action is given by $S_{imp} = \int d^2x V(x)|z_\sigma|^2$[18]. Here $V(x)$ is a random potential resulting from quenched disorder. The random potential is random only in space but static in time, i.e., $V(x) = V(r)$. We assume that $V(r)$ is a gaussian random potential with $< V(r)V(r') >= W\delta(r-r')$ and $< V(r) >= 0$. Integrating over the massive U(1) gauge field a_μ and using the standard replica trick to average over the gaussian random potential, we obtain an effective action in the presence of non-magnetic disorder

$$S_{eff} = \int drd\tau \sum_{\alpha=1}^{N}\Big[\frac{1}{2g}|\partial_\mu z_{\sigma\alpha}|^2 + m^2|z_{\sigma\alpha}|^2 + \frac{u}{2}(|z_{\sigma\alpha}|^2)^2\Big] - \frac{W}{2}\int drd\tau_1 d\tau_2 \sum_{\alpha,\beta=1}^{N}|z_{\sigma\alpha}(\tau_1)|^2|z_{\sigma\beta}(\tau_2)|^2. \quad (5)$$

Here α, β are replica indices and the limit $N \to 0$ is to be taken at the end. In the above a local current interaction originating from integration over the massive gauge field a_μ is not explicitly taken into account because it is expected to be irrelevant (marginally) in the low energy limit. From this effective action one can easily see a fate of the spinon. It is well known that the non-magnetic disorder results in localization of massive particles in $(1+1)D$[11, 19]. The spinon is trapped by the disorder. The non-magnetic disorder is expected to act as magnetic disorder owing to the trapped spinon.

Next we consider a two dimensional doped $U1SL$ which is suggested to describe physics of the pseudogap phase[9, 10]. The problem of hole doping to the $U1SL$

is examined in the context of a SU(2) slave boson theory developed by Lee, Wen, and coworkers[20]. Following Wen and Lee, we consider the staggered flux phase as an ansatz for the pseudogap state. In the staggered flux phase a spin degree of freedom is described by QED_3 in terms of the massless Dirac spinons interacting via compact U(1) gauge fields. Doped holes are represented by holons carrying only a charge degree of freedom[21]. In the SU(2) slave boson theory the holons have two components in association with the SU(2) symmetry[22]. An effective Lagrangian for the holon field is given by a non-linear σ model coupled to the spinons via the compact U(1) gauge field[20]. The problem of hole doing to the $U1SL$ is investigated by an effective Lagrangian in the staggered flux phase[12]

$$\begin{aligned} Z &= \int D\psi_\alpha Dz_\beta Da^3_\mu e^{-\int d^3x \mathcal{L}}, \\ \mathcal{L} &= \bar{\psi}_\alpha \gamma_\mu (\partial_\mu \delta_{\alpha\beta} - \frac{i}{2} a^3_\mu \tau^3_{\alpha\beta})\psi_\beta \\ &+ \frac{1}{2g}|(\partial_\mu \delta_{\alpha\beta} - \frac{i}{2} a^3_\mu \tau^3_{\alpha\beta}) z_\beta|^2 + V(z_\alpha) \\ &+ G\bar{\psi}_\alpha \tau^k_{\alpha\beta}\psi_\beta z^\dagger_\gamma \tau^k_{\gamma\delta} z_\delta + \frac{1}{2e^2_{in}}|\partial \times a^3|^2. \end{aligned} \tag{6}$$

Here ψ_α is a 4 component massless Dirac fermion with an isospin index $\alpha = 1, 2$, and z_α, a phase field of a holon doublet[22]. $V(z_\alpha)$ is an effective potential for easy plane anisotropy, resulting from contributions of high energy fermions[12, 20]. $g^{-1} \sim x$ is a phase stiffness of the holon field with hole concentration x, and $G \sim x$, a coupling constant between spinon and holon isospins. τ^k acts on SU(2) isospin space. e_{in} is an effective internal gauge charge. The spinons and holons interact via not only the gauge field a^3_μ but also their isospins. The coupling between the spinon and holon isospins is expected to result from gauge interactions mediated by the time component of the SU(2) gauge fields[12]. Similar consideration can be found in Ref. [20]. The kinetic energy of the gauge field results from particle-hole excitations of high energy quasiparticles. Utilizing this effective Lagrangian, recently we showed that antiferromagnetism (AFM) can coexist with $d - wave$ superconductivity (dSC)[12]. The AFM results from the Dirac fermions and the dSC, holon condensation. Holon condensation is shown to result in a zero mode of a nodal fermion (spinon) in a single instanton potential[12]. Thus instanton excitations are suppressed. The coupling between the spinon and holon isospins is very crucial for existence of the zero mode. If the term is ignored, the fermion zero mode is not found[24]. Since the coupling constant G is proportional to hole concentration x, there exists no fermion zero mode at half filling. Suppression of instanton excitations resulting from the fermion zero mode leads to deconfinement of the spinons and holons at $T = 0$ K in underdoped superconducting phase. Further, an instanton in the presence of the fermion zero mode induces a fermion mass[12]. The mass results in AFM[23, 24]. Thus the AFM of the nodal fermions is expected to coexist with the dSC in the underdoped cuprates. Despite the coexistence the AFM has little effect on the dSC[12]. As a result a superconductor to insulator transition in the underdoped region is found to fall in the XY universality class consistent with experiments[25].

Now we derive an effective action for the massive Dirac fermions as we do Eq. (5) in $(1+1)D$. Performing a standard duality transformation of the non-linear σ model for the holon fields[12, 15], we obtain

$$\begin{aligned} S &= \int d^3x \Big[|(\partial_\mu - ic_{\uparrow\mu})\Phi_\uparrow|^2 + |(\partial_\mu - ic_{\downarrow\mu})\Phi_\downarrow|^2 \\ &+ V(|\Phi_\uparrow|, |\Phi_\downarrow|) + \frac{1}{2\rho}|\partial \times c_\uparrow|^2 + \frac{1}{2\rho}|\partial \times c_\downarrow|^2 \\ &+ \frac{1}{2e^2_{in}}|\partial \times a^3|^2 - i(\partial \times a^3)_\mu (c_{\uparrow\mu} - c_{\downarrow\mu}) \\ &+ \bar{\psi}_\alpha \gamma_\mu (\partial_\mu \delta_{\alpha\beta} - \frac{i}{2} a^3_\mu \tau^3_{\alpha\beta})\psi_\beta + m_\psi \bar{\psi}_\alpha \psi_\alpha\Big]. \end{aligned} \tag{7}$$

Here $\Phi_{\uparrow(\downarrow)}$ is a vortex field with isospin $\uparrow$ $(\downarrow)$ and $c_{\uparrow(\downarrow)\mu}$, its corresponding vortex gauge field mediating interactions between the vortices. $V(|\Phi_\uparrow|, |\Phi_\downarrow|)$ is an effective potential including vortex mass and self-interactions. $\rho \sim g^{-1} \sim x$ originates from the phase stiffness with the hole concentration x. m_ψ is a mass of the nodal fermion. Suppression of instantons originating from the fermion zero mode leads the U(1) gauge field a^3_μ to be non-compact. In the superconducting state the vortex fields are not condensed. As a result the vortex gauge fields remain massless. Integrating over the massless vortex gauge fields, we obtain a massive U(1) gauge field. We note that in one dimension integration over the ϕ_- field results in the massive gauge field. Integrating over the massive internal gauge field, we obtain an effective action for the massive spinons which is essentially same as Eq. (5) (in the absence of the disorder)

$$S_{eff} = \int d^3x \Big[\bar{\psi}_\alpha \gamma_\mu \partial_\mu \psi_\alpha + m_\psi \bar{\psi}_\alpha \psi_\alpha\Big]. \tag{8}$$

Here a local current interaction term $\frac{1}{\rho}|\bar{\psi}_\alpha \gamma_\mu \psi_\alpha|^2$ generated by integration over the massive gauge field is irrelevant in the low energy limit and thus can be safely ignored. In summary, holon condensation in the doped $U1SL$ leads a spin degree of freedom to be a gapped $U1SL$ and a charge degree of freedom to be superconducting. This is essentially same as the one dimensional case except the fact that the spinon is a fermion here while it is a boson in one dimensional case.

Now we examine a role of the non-magnetic impurities in the U(1) spin liquid with the massive spinons. An impurity action is given by[11]

$$S_{imp} = \int d^3x V(x) \bar{\psi}_\alpha \gamma_0 \psi_\alpha. \tag{9}$$

Here $V(x)$ is a random potential as discussed in one dimension. It does not depend on time, i.e., $V(x) = V(\mathbf{r})$.

We also assume that $V(\mathbf{r})$ is a gaussian random potential with $< V(\mathbf{r})V(\mathbf{r}') >= W\delta(\mathbf{r}-\mathbf{r}')$ and $< V(\mathbf{r}) >= 0$. Using the standard replica trick to average over the gaussian random potential, we obtain an effective action in the presence of the non-magnetic disorder

$$S_{eff} = \int d^2\mathbf{r}d\tau \sum_{k=1}^{M} \Big[\bar{\psi}_{\alpha k}\gamma_{\mu}\partial_{\mu}\psi_{\alpha k} + m_{\psi}\bar{\psi}_{\alpha k}\psi_{\alpha k}\Big] - \frac{W}{2}\int d^2\mathbf{r}d\tau_1 d\tau_2 \sum_{k,l=1}^{M} (\bar{\psi}_{\alpha k}\gamma_0\psi_{\alpha k})(\tau_1)(\bar{\psi}_{\alpha l}\gamma_0\psi_{\alpha l})(\tau_2). \quad (10)$$

Here k, l are replica indices and the limit $M \rightarrow 0$ is to be taken at the end. This effective action is essentially same as Eq. [5] except the fact that the spinons are fermions here. It is well known that the massive Dirac fermion is localized by the non-magnetic impurities in $(2+1)D$[11]. The non-magnetic impurity is expected to act as a magnetic impurity.

We believe that the localized magnetic moment is not screened by other spinons. This is because the spinons are massive and thus there are no density of states at the Fermi energy. As a result the localized magnetic moment is expected to act as a free spin. We think that this is the origin to cause a Curie-Weiss behavior of a magnetic susceptibility[1, 3, 5] and enhancement of antiferromagnetic correlations[2]. But we admit that our conclusion is based on $T = 0$ K calculation. It is not clear whether our results hold at $T \neq 0$ K. This is because an instanton effect is not clearly understood at finite temperature. The role of non-magnetic disorder in a doped $U1SL$ at finite temperature is left for future study.

Hole doping to a Mott insulator leads a critical spin liquid to a gapped spin liquid in both one and two dimensions. We examined the role of non-magnetic impurities in the gapped spin liquid. We showed that the massive spinon is localized in the non-magnetic disorder. The localized magnetic moment in the non-magnetic disorder is expected to explain the Curie-Weiss behavior of a magnetic susceptibility[1, 3, 5].

K.-S. Kim thanks Dr. Park, Kee-Su for introducing Ref. [17].

[1] N. Nagaosa and T. K. Ng, Phys. Rev. B **51**, 15588 (1995); references therein; N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **79**, 3755 (1997); references therein.

[2] C. Pepin and P. A. Lee, Rev. Lett. **81**, 2779 (1998); references therein.

[3] G. Khaliullin et al., Phys. Rev. B **56**, 11882 (1997); references therein; R. Kilian et al., Phys. Rev. B **59**, 14432 (1999); references therein.

[4] S. Sachdev and M. Vojta, cond-mat/0009202; references therein; A. Polkovnikov et al., Phys. Rev. Lett. **79**, 3755 (1997); references therein.

[5] M. Vojta and R. Bulla, Phys. Rev. B **65**, 014511 (2001); references therein.

[6] S. Sachdev, Rev. Mod. Phys. **75**, 913 (2003).

[7] Chetan Nayak, Phys. Rev. Lett. **85**, 178 (2000).

[8] Dung-Hai Lee, Phys. Rev Lett **84**, 2694 (2000).

[9] M. Hermele et al., cond-mat/0404751.

[10] T. Senthil and Patrick A. Lee, cond-mat/0406066.

[11] Ki-Seok Kim, cond-mat/0407151; Phys. Rev. B (R) accepted; references therein.

[12] Ki-Seok Kim, cond-mat/0408236; Phys. Rev. Lett. submitted; references therein.

[13] N. Nagaosa, Quantum Field Theory in Strongly Correlated Electronic Systems, Springer (1999).

[14] S. Sachdev and K. Park, Annals of Phys. **298**, 58 (2002).

[15] T. Senthil et al., Science **303**, 1490 (2004); T. Senthil et al., cond-mat/0311326; references therein.

[16] R. Shankar, Phys. Rev. Lett. **63**, 203 (1989); R. Shankar, Nucl. phys. B **330**, 433 (1990).

[17] Edward Witten, Nucl. phys. B **149**, 285 (1979).

[18] An impurity Hamiltonian is given by

$$H_{imp} = \sum_{i=1}^{N} V_i c_{\sigma i}^{\dagger} c_{\sigma i}.$$

Using the slave-fermion constraint $\sum_{\sigma=1}^{2} |z_{\sigma i}|^2 + f_i^{\dagger} f_i = 1$ with a fermionic holon f_i at site i, we find that the electron density is given by the spinon density, i.e., $c_{\sigma i}^{\dagger} c_{\sigma i} = |z_{\sigma i}|^2$. This is proved by

$$c_{\sigma i}^{\dagger} c_{\sigma i} = z_{\sigma i}^{\dagger} z_{\sigma i} f_i f_i^{\dagger} = (1 - f_i^{\dagger} f_i) f_i f_i^{\dagger} = (1 - f_i^{\dagger} f_i)(1 - f_i^{\dagger} f_i) = 1 - 2 f_i^{\dagger} f_i + f_i^{\dagger} f_i f_i^{\dagger} f_i = 1 - 2 f_i^{\dagger} f_i + f_i^{\dagger} f_i = 1 - f_i^{\dagger} f_i = z_{\sigma i}^{\dagger} z_{\sigma i}.$$

[19] I. F. Herbut, Phys. Rev. Lett. **87**, 137004 (2001); references therein.

[20] P. A. Lee et al., Phys. Rev. B **57**, 6003 (1998); P. A. Lee and N. Nagaosa, Phys. Rev. B **68**, 024516 (2003); J. Kishine et al., Phys. Rev. B **65**, 064526 (2002).

[21] In the one dimensional doped Mott insulator we used a slave-fermion representation. Here a spin degree of freedom is represented by a bosonic spinon and a charge degree of freedom, a fermionic holon. But in the two dimensional doped Mott insulator we used a slave-boson representation. In this case a spin degree of freedom is represented by a fermionic spinon and a charge degree of freedom, a bosonic holon.

[22] In the SU(2) slave boson theory a local SU(2) symmetry originates from the symmetry of a spin $\uparrow$ spinon and a spin $\downarrow$ anti-spinon at half filling[20]. As a result of the SU(2) symmetry many U(1) ground states such as a staggered flux phase, $d-wave$ pairing state, and rotated states between the two are exactly degenerate at half filling[20]. In underdoped region the U(1) ground states are expected to remain nearly degenerate. This motivates Wen and Lee to extend the SU(2) symmetry at half filling away from half filling[20]. In the SU(2) slave boson theory fluctuations between the nearly degenerate U(1) states can be easily taken into account via introduction of the holon doublets[20].

[23] D. H. Kim and P. A. Lee, Annals of Phys. **272**, 130 (1999); references therein.

[24] J. B. Marston, Phys. Rev. Lett. **64**, 1166 (1990).
[25] Ki-Seok Kim, Jae-Hyeon Eom, Young-Il Seo, and Sung-Ho Suck Salk, Phys. Rev. B **69**, 014504 (2004); references therein; Ki-Seok Kim, Sung-Sik Lee, Jae-Hyeon Eom, and Sung-Ho Suck Salk, cond-mat/0404527; Phys. Rev. Lett. submitted; references therein.